Key.



**Think About It**

List some similarities and differences between a rectangle and a square

| **Similarities** | **Differences** |
| --- | --- |
| - 4 sides<br>- 4 right angles (90°)<br>- both are quadrilaterals | - rectangles have different side lengths (2)<br>- squares have all the same side lengths. |

**Investigate** - model with cubes before you draw
- organize your work neatly.

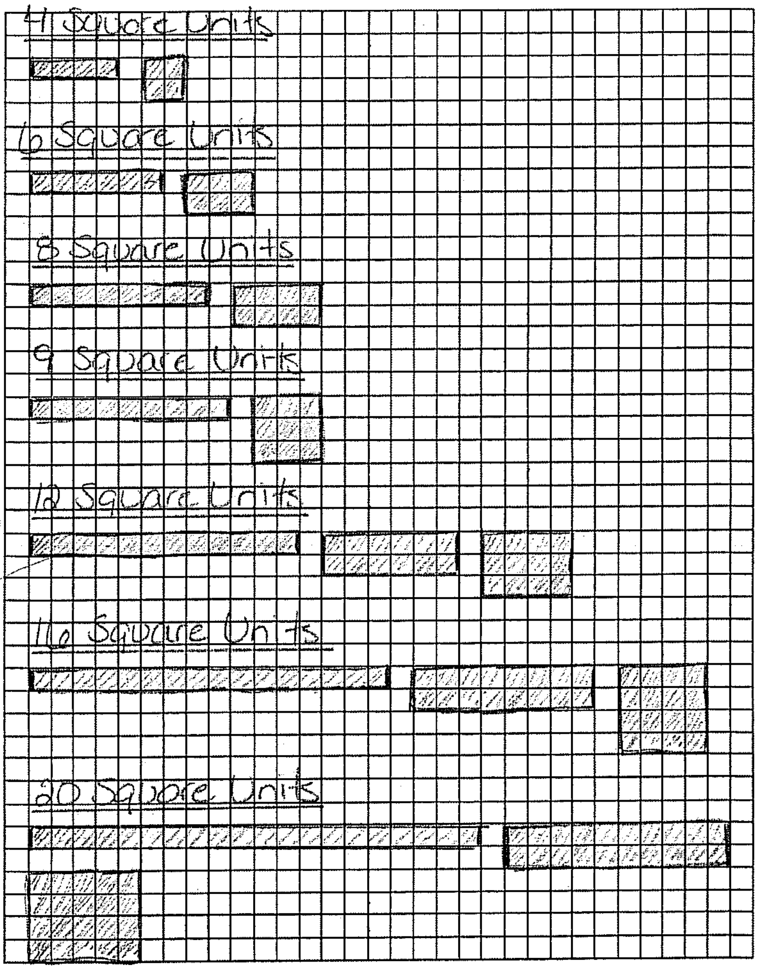
Use the provided grid and 20 square cubes to make as many different rectangles as you can with the areas listed below. Draw each of your rectangles on the grid paper.

4 square units
12 square units

9 square units
20 square units

16 square units
8 square units

6 square units




1. For how many of the areas were you able to make a square?
   3 areas: 4, 9 and 16

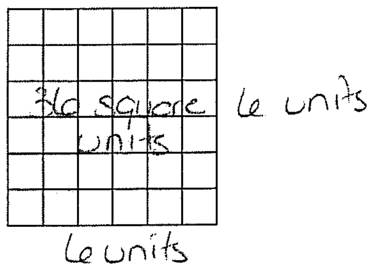
2. What are the lengths of the sides of each square that you made?
   4 square units = 2 × 2
   9 square units = 3 × 3
   16 square units = 4 × 4

3. How is the side length of the square related to its area?
   area = side × side
   or
   The side length of a square multiplied by itself equals the area

4. Name two areas greater than 20 square units for which you could use the cubes to make a square. How do you know you could make a square for each of these areas?
   25, 36, 49 etc.
   we know this because
   5 × 5 = 25
   6 × 6 = 36
   7 × 7 = 49

**Connect**

When we multiply a number by itself, we square the number.




What is the square of 6? $6 \times 6 = 36$

We can write this as: $6^2 = 36$

We say: six squared is thirty-six

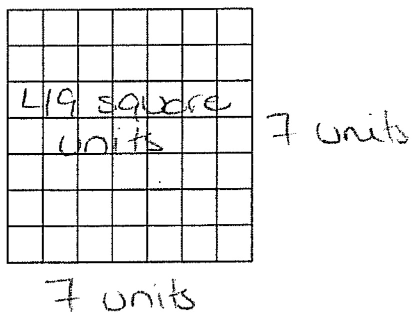
36 is an example of a square number, or a perfect square.

A square with an area of 36 square units, will have side lengths of 6 units.

We can model a square number by drawing a square whose area is equal to the square number

**Example 1: Show that 49 is a square number. Use a diagram, symbols, and words.**

Draw a square.




The area of the square is: 49 square units

The side length of the square is: 7 units

We say: forty-nine is seven squared

*Extension: Is the number 1 a square number? How can you tell?*

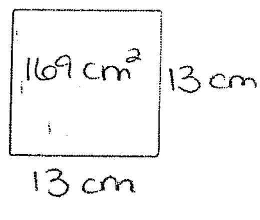
yes, a square with side length 1 unit has an area of 1 square unit since $1 \times 1 = 1$

**Example 2: A square picture has an area of 169 cm$^2$. Find the perimeter of the picture.**

- The picture is a square with area 169 cm$^2$
- Find the side length of the square (side length is required to find perimeter)
- what number when multiplied by itself gives 169?

$13 \times 13 = 169$

So, the picture has side length 13 cm.




- Perimeter is the distance around the picture

$P = 13 + 13 + 13 + 13$
$= 52$ cm

$\therefore$ The perimeter of the picture is 52 cm.

*Extension: Could you find the area of a square given its perimeter? How?*

yes, since each side of a square is identical, and you have 4 sides, you could divide the perimeter by 4 to find the side length. Then, multiply the side length by itself to find the area

eg. Perimeter is 12 cm ; side length $12 \div 4 = 3$ cm $\therefore$ Area $= 3 \times 3 = 9$ cm$^2$

Key.



### Think About It

Recall, a square number is the product of a number multiplied by itself. For example, 9 is a square number because 3 x 3 = 9.

A factor is a number that divides exactly into another number.

What are the factors of 10?

1, 2, 5 and 10.

What are the ways you can write 10 as a product of these factors?

1 × 10 , 2 × 5

### Investigate

The following chart shows the factors of each whole number from 1 to 8. Complete the chart by finding the factors for the remaining numbers. Write the factors in ascending order. Circle any factors that occur twice.

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | | | | 24 | | | | | | 30 |
| | | | | | | | | | | | | | | | | | | | | | | | 12 | | | | | | 15 |
| | | | | | | | | | | | 12 | | | | | | 18 | | 20 | | | | 8 | | | | 28 | | 10 |
| | | | | | | | | | | | 6 | | | | 16 | | 9 | | 10 | | | | 6 | | | | 14 | | 6 |
| | | | | | 6 | | 8 | | 10 | | 4 | | 14 | 15 | 8 | | 6 | | 5 | 21 | 22 | | 4 | | 26 | 27 | 7 | | 5 |
| | | | 4 | | 3 | | 4 | 9 | 5 | | 3 | | 7 | 5 | 4 | | 3 | | 4 | 7 | 11 | | 3 | 25 | 13 | 9 | 4 | | 3 |
| | 2 | 3 | 2 | 5 | 2 | 7 | 2 | 3 | 2 | 11 | 2 | 13 | 2 | 3 | 2 | 17 | 2 | 19 | 2 | 3 | 2 | 23 | 2 | 5 | 2 | 3 | 2 | 29 | 2 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |

1. Which numbers have an odd number of factors?

   1, 4, 9, 16, 25 – all perfect squares, only numbers with an odd number of factors

2. Which numbers in this chart are square numbers? How do you know? Do you notice a pattern with the repeated factors of square numbers?

   1, 4, 9, 16, 25 – Because one of their factors is repeated
   – Repeated factors are consecutive whole numbers, always middle factor.

3. What is a square root? How is squaring a number related to taking a square root of a number?

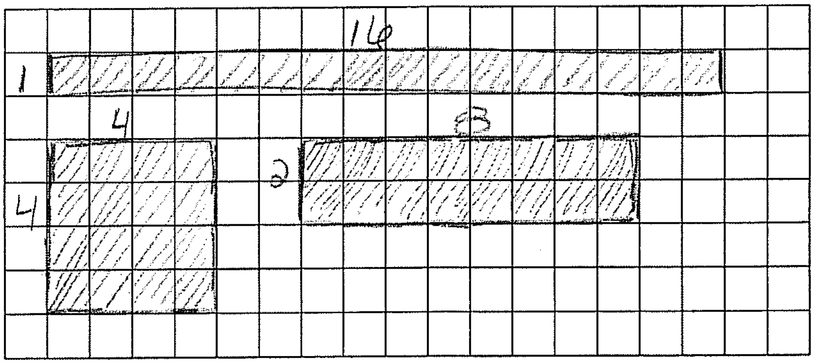
   They are inverse operations, ie. they undo each other

   ie. $4 \times 4 = 4^2 = 16 \Longleftrightarrow \sqrt{16} = \sqrt{4 \times 4} = \sqrt{4^2} = 4$

## Connect

We can use factoring to determine if a given number is a square number:

- Square numbers ALWAYS have an odd number of factors since factors occur in pairs because they represent the dimensions of the rectangles we can make using the given number. Take a look at the number 16.




Sixteen has 5 factors: 1, 2, 4, 8 and 16

Since there are an odd number of factors, one rectangle is a square

The square has a side length of 4.

We say that 4 is a square root of 16.

We write: $\sqrt{16} = 4$.

16 is an example of a perfect square, or a square number.

* Perfect squares have square roots that are whole numbers!

**Example 1: Find the square of each number.**

**a) 5** $5^2 = 5 \times 5$
$= 25$

**b) 15** $15^2 = 15 \times 15$
$= 225$

**Example 2: Evaluate**

**a) $8^2$** $= 8 \times 8$
$= 64$

**b) $3^2$** $= 3 \times 3$
$= 9$

**c) $7^2$** $= 7 \times 7$
$= 49$

**Example 3: Find the square root of the following**

**a) 64**
$\sqrt{64} = \sqrt{8 \times 8}$
$= 8$

**b) 25**
$\sqrt{25} = \sqrt{5 \times 5}$
$= 5$

**c) 100**
$\sqrt{100} = \sqrt{10 \times 10}$
$= 10$

**Example 4: Is 136 a perfect square? How do you know?**

Factor pairs
$1 \times 136$
$2 \times 68$
$4 \times 34$
$8 \times 17$

repeated $\left\{ \begin{array}{l} 17 \times 8 \\ 34 \times 4 \end{array} \right.$
etc.

∴ Factors are 1, 2, 4, 8, 17, 34, 68, 136
There are an even number of factors and no repeated factors so 136 is not a square number

OR

$11 \times 11 = 121$
$12 \times 12 = 144$ ← 136 is between these two square numbers so it cannot have a whole number square root

**Math 8**
Square Roots and the Pythagorean Theorem
1.3/1.4 Quiz

Key. /10

**Unit 1 – Quiz 2**

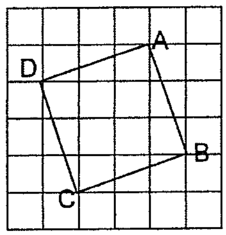
**Use any method you would like to answer the following questions. Show ALL work!!**

1. In your own words, define what a "square number" is?

/2

→ product of a number multiplied by itself
→ a number with an odd number of factors because one factor is repeated
– a number whose square root is a whole number
– can be modelled as a square where Area is equal to side × side or $(\text{side})^2$

2. Find the area and the side length of the square ABCD.

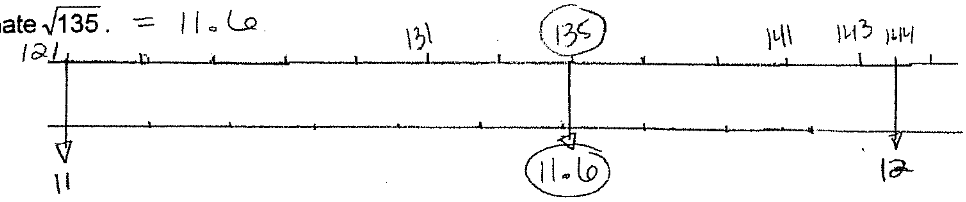



/4

$$\text{Area} = A_{\square} + 4A_{\triangle}$$
$$= 4 + 4\left(\frac{(1)(3)}{2}\right)$$
$$= 4 + 6$$
$$= 10 \text{ units square}$$
$$\text{side length} = \sqrt{10}$$

3. Estimate $\sqrt{135}$. $= 11.6$

/4



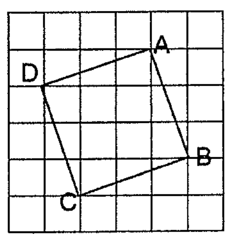

**Math 8**
Square Roots and the Pythagorean Theorem
1.3/1.4 Quiz

**Unit 1 – Quiz 2**

**Use any method you would like to answer the following questions. Show ALL work!!**

1. In your own words, define what a "square number" is?

2. Find the area and the side length of the square ABCD.



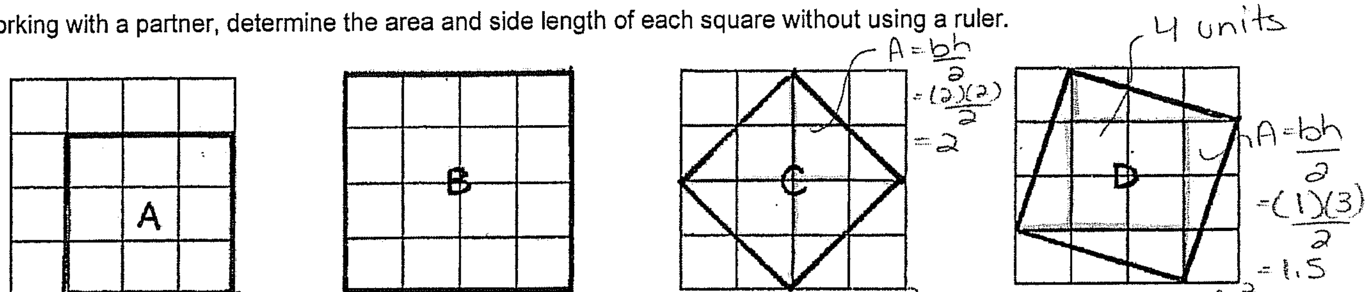

3. Estimate $\sqrt{135}$.

Key.



## Investigate

Working with a partner, determine the area and side length of each square without using a ruler.




Area: 9 $units^2$ Length: 3 units

Area: 16 $units^2$ Length: 4 units

Area: 8 $units^2$ Length: $\sqrt{8}$ units.

Area: 10 $units^2$ Length: $\sqrt{10}$ units

1. Describe any patterns in your measurements.

Always take the square root of the area to find the side length.

2. How did you find the area and side length of each square?

- Counted the number of whole and part squares
- Found the side length and multiplied it by itself to find area.
- Broke square into shapes and added the areas of all the shapes together

3. How did you write the side lengths of squares C and D?

Wrote them with square root symbols (recall $\sqrt{8} \times \sqrt{8} = 8$)

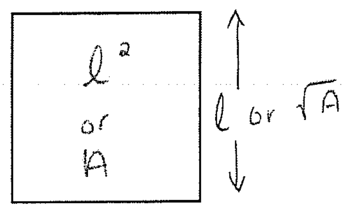
## Connect

We can use the properties of a square to find its area or side length.

Area of a square = length × length = $length^2$

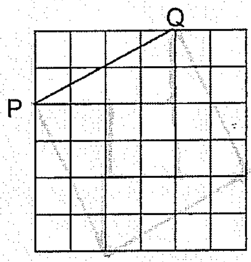
When the side length is $l$, the area is $l^2$

When the area is $A$, the side length is $\sqrt{A}$.




We can calculate the length of any line segment on a grid by thinking of it as the side length of a square.

**Example 1: Find the length of line segment PQ.**




- Use the line segment to construct a square because if we can find area, we can find side length ($\sqrt{A} = l$)
- Break square into shapes that you can find the area of (start with $\Delta$'s)
- Sum up the areas of the shapes.

$A_{\square} = 4 \text{ units}^2$

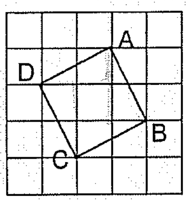
$A_{\Delta} = \frac{bh}{2}$
$= \frac{(2)(4)}{2}$
$= 4 \text{ units}^2$

$A_{TOTAL} = A_{\square} + A_{\Delta's}$
$= 4 + 4 + 4 + 4 + 4$
$= 20 \text{ units}^2$

Length of side $= \sqrt{A} = \sqrt{20}$ units

$\therefore$ Line segment PQ is $\sqrt{20}$ units long

**Example 2: Find the area of square ABCD. What is the length of side AB?**




$A_{\square} = 1 \text{ unit}^2$

$A_{\Delta} = \frac{bh}{2}$
$= \frac{(1)(2)}{2}$
$= 1 \text{ unit}^2$

$\therefore A_{TOTAL} = 1 + 1 + 1 + 1 + 1$
$= 5 \text{ units}^2$

side AB $= \sqrt{A} = \sqrt{5}$ units

$\therefore$ side AB is $\sqrt{5}$ units long

*Extension: Is the area of every square a square number?*

No, the area of every square is <u>NOT</u> a square number
eg. 20 is not a square number because we can not write $\sqrt{20}$ as a whole number

*Extension: When will the length of a line segment not be a whole number?*

The length of a line segment will not be a whole number when the area of the square is not a square number

Key



### Think About It

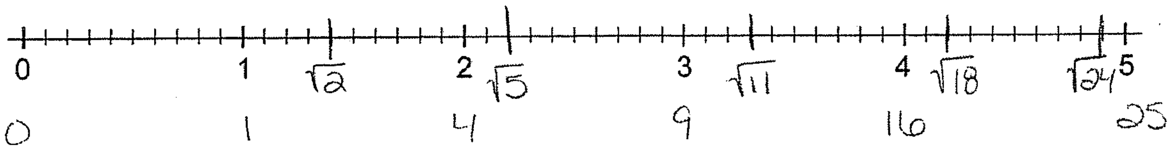
You know that a square root of a given number is a number which when multiplied by itself equals the given number eg. $\sqrt{9} = \sqrt{3 \times 3} = 3$

You also know that the square root of a number is the side length of a square with an area that is equal to the given number.

$\boxed{9}$ 3 (side lengths 3 and 3)

### Investigate

Use the number line below, place each square root on the number line to show its approximate value: $\sqrt{2}$, $\sqrt{5}$, $\sqrt{11}$, $\sqrt{18}$, $\sqrt{24}$. Do not use a calculator!!




1. What strategies did you use to estimate the square roots?
   - Comparison to square roots of perfect squares
     eg $\sqrt{2}$ is between $\sqrt{1} = 1$ and $\sqrt{4} = 2$, but closer to $\sqrt{1}$

2. How could you use a second number line showing square numbers?
   - To more quickly identify the square numbers it is between.

3. How could you use a calculator to check your answers?
   - Trial and error – multiply the number by itself, refining as you go, until your answer nearly equals the given number you are trying to find the square root of.
   - Use the $\sqrt{\ }$ key on the calculator

4. Is 4.8 or 4.9 a better estimate for $\sqrt{24}$? Why? How could you get an even more accurate answer?

   $4.8 \times 4.8 = 23.04$
   $4.9 \times 4.9 = 24.01$ ← better estimate because it is closer to 24
   $\therefore \sqrt{24} \doteq 4.9$

   You could get a more accurate answer by having more decimals.

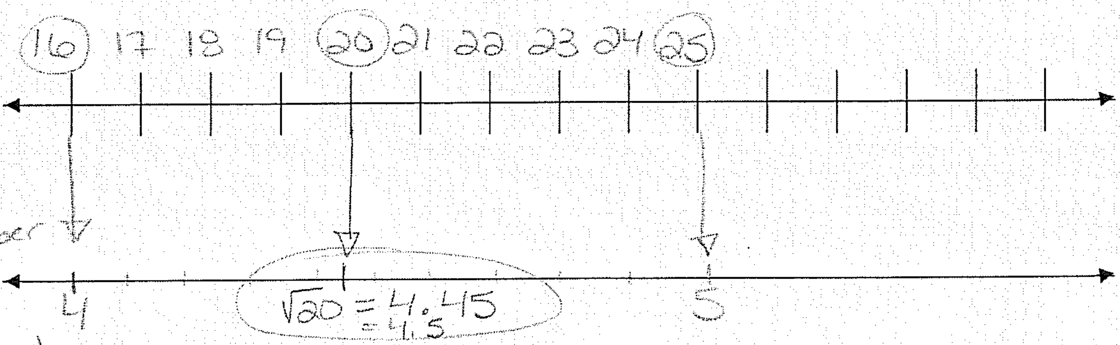
## Connect

**Example 1: Estimate $\sqrt{20}$ using two number lines.**

1) Find the perfect squares 20 is between (use multiplication table)

$16 < 20 < 25$

Place this information on 1st number line




2) Take $\sqrt{\ }$ and place on 2nd number line

3) divide 2nd number line into 10 equal segments

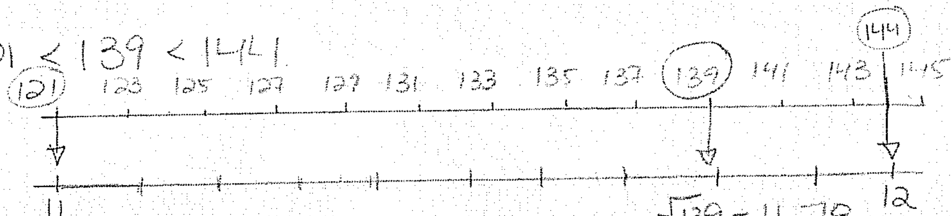
4) estimate answer

**Example 2: A square garden has an area of 139 m².**

**A) What are the approximate dimensions of the garden to one decimal place?**

**B) How much fencing would be needed to surround the garden?**

A) $121 < 139 < 144$




$\sqrt{139} = 11.78$

$\sqrt{139} = 11.8$

$\therefore$ The side length of the garden is 11.8 m.

B) Perimeter $= 11.8 + 11.8 + 11.8 + 11.8$

$= 47.2$ m ← round up for purchasing

or $P = 4 \times 11.8$
$= 47.2$ m

$\therefore$ 48 m of fencing should be purchased.

***Extension:* How do square numbers help us estimate square roots?**

They provide benchmarks that can be used to determine which two whole numbers a square root is between

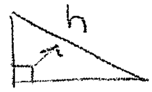
Key



**Think About It**

What is a right triangle? Draw a right triangle.

Any triangle that contains a right angle (90°) ie.




How can you identify the hypotenuse of a right triangle? What is special about the hypotenuse?

The hypotenuse is always across from the right angle. It is special because it is the longest side.

How can you identify the legs of a right triangle?

They form an "L" shape on either side of the right angle.

**Investigate**

With a partner:

1. Construct a square off of each line segment that forms triangle ABC.

2. Find the area and the side length of each square. Show your work. Label the areas and the side lengths on your diagram.

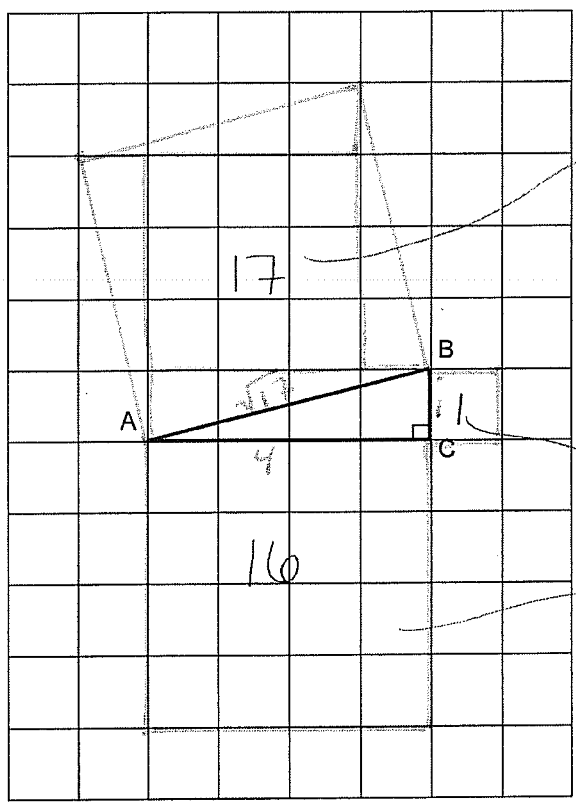
3. What relationship do you see among the areas of the squares on the sides of a right triangle?

   sum of the areas of the legs = area of the hypotenuse

   the sum of the squares of the legs = square of the hypotenuse

4. Is there a relationship among the side lengths of the right triangle?

   No, the relationship is between the areas!




WORK:

$$A = A_{\square} + 4A_{\triangle}$$
$$= 9 + 4\left(\frac{bh}{2}\right)$$
$$= 9 + 4\left(\frac{1 \times 4}{2}\right)$$
$$= 9 + 4(2)$$
$$= 9 + 8$$
$$= 17 \text{ units}^2$$

$A = 1 \text{ unit}^2$

$A = 16 \text{ units}^2$

$$\underbrace{1 + 16}_{\text{areas of the legs}} = \underbrace{17}_{\text{area of the hypotenuse}}$$

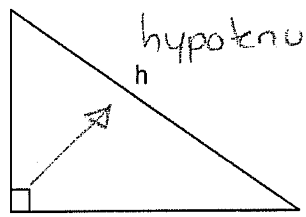
or $\text{leg } a^2 + \text{leg } b^2 = \text{hypotenuse}^2$

## Connect

In all right triangles, the area of the square of the hypotenuse is equal to the sum of the areas of the squares of the legs.

This relationship is called the Pythagorean Theorem, and it is used to find the length of any side when we know the lengths of the other two sides.

It can be written algebraically as:



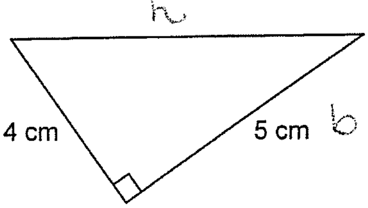

$$h^2 = a^2 + b^2$$

the square of the hypotenuse = the sum of the squares of the legs

*** Make sure you label your triangle correctly!!!

**Example 1: Find the length of the unknown side. Give your answer to one decimal place (use a calculator).**

**A)**




$h^2 = a^2 + b^2$

$h^2 = 4^2 + 5^2$

$h^2 = 16 + 25$

$h^2 = 41$ ← this is the area, we want side length

$\sqrt{h^2} = \sqrt{41}$

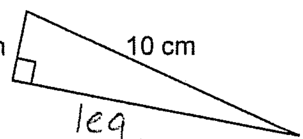
$h = \sqrt{41}$ cm

or

$h = 6.4$ cm

∴ The hypotenuse is 6.4 cm long.

**B)**




$h^2 = a^2 + b^2$

$10^2 = 3^2 + b^2$

$100 = 9 + b^2$

$-9 \quad -9$

$91 = b^2$

$\sqrt{91} = \sqrt{b^2}$

$\sqrt{91}$ cm $= b$

or

$9.5$ cm $= b$

Could show:

$10^2 - 3^2 = b^2$

$100 - 9 = b^2$

$91 = b^2$

$\sqrt{91} = b$

∴ The leg is 9.5 cm long

Assigned: p. 43 #3-12, 14



Key.

**Connect**

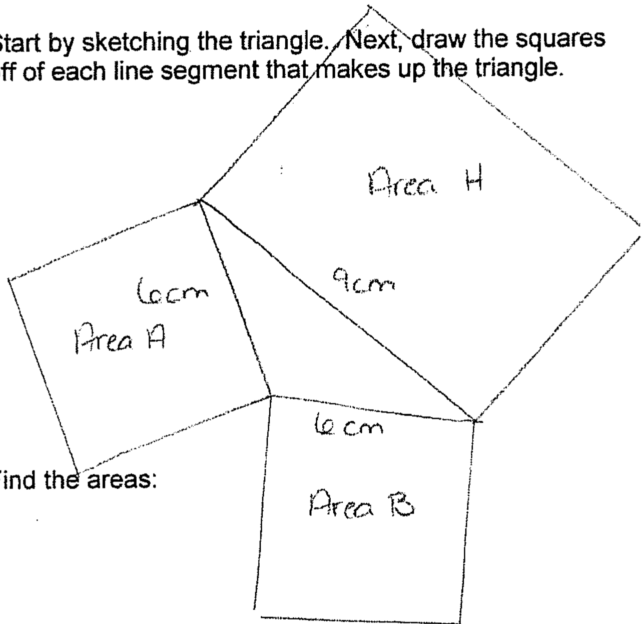
The Phythagorean Theorem applies ONLY to ____________ triangles! We can use the Pythagorean Theorem to prove whether or not a triangle is a right triangle.

A set of 3 whole numbers that satisfies the Pythagorean Theorem is called a ______________________________.

**Example 1: Determine whether each triangle with the given side lengths is a right triangle.**

**A) 6 cm, 6 cm, 9 cm**

Start by sketching the triangle. Next, draw the squares off of each line segment that makes up the triangle.




Find the areas:

Area A = $6 \times 6 = 36\ cm^2$

Area B = $6 \times 6 = 36\ cm^2$

Area H = $9 \times 9 = 81\ cm^2$

Verify the relationship:

Pythagorean Theorem says:

$$h^2 = a^2 + b^2$$

or

area of hypotenuse = sum of areas of legs

so if this is a right triangle then,

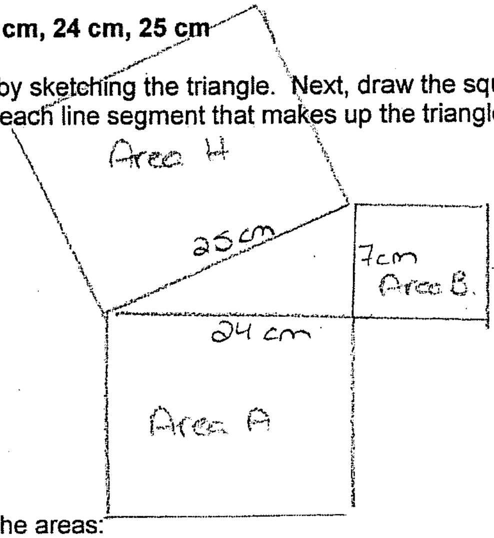
$$81 = 36 + 36$$

$$81 \neq 72$$

∴ Triangle is not a right triangle

**B) 7 cm, 24 cm, 25 cm**

Start by sketching the triangle. Next, draw the squares off of each line segment that makes up the triangle.




Find the areas:

Area A = $24 \times 24 = 576\ cm^2$

Area B = $7 \times 7 = 49\ cm^2$

Area H = $25 \times 25 = 625\ cm^2$

Verify the relationship:

$$h^2 = a^2 + b^2$$

$$625 = 576 + 49$$

$$625 = 625\ ✓$$

∴ Triangle is a right triangle and 7-24-25 is a Pythagorean Triple.

Assignment p. 49 # 4-14, 16, 21, 22

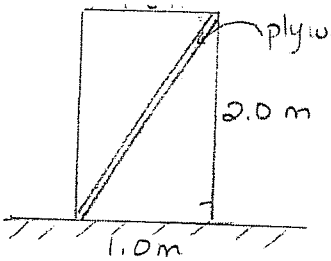


Key

### Connect

We can use the Pythagorean Theorem to solve problems that involve right triangles. When solving these types of problems it is best to start by drawing a picture of the problem and labeling any information you know.

**Example 1: A doorway is 2.0 m high and 1.0 m wide. A square piece of plywood has side length 2.2 m. Can the plywood fit through the door?**




plywood - does it fit? ∴ Hypotenuse must be 2.2 m or longer so the plywood will fit.

Use Pythagorean Theorem:

$h^2 = a^2 + b^2$

$h^2 = (1.0)^2 + (2.0)^2$

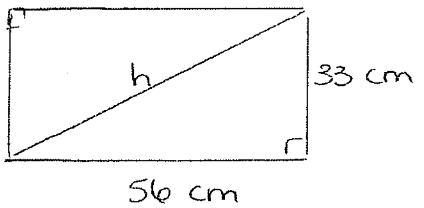
$h^2 = 1 + 4$

$\sqrt{h^2} = \sqrt{5}$

$h = 2.24$ m

∴ The plywood will fit through the doorway!

**Example 2: Marina helped her dad build a small rectangular table for her bedroom. The tabletop has length 56 cm and width 33 cm. The diagonal of the tabletop measures 60 cm. Does the tabletop have square corners?**




Suppose the diagonal of the tabletop is the hypotenuse of a right triangle, then using Pythagorean Theorem

$h^2 = a^2 + b^2$

$h^2 = (56)^2 + (33)^2$

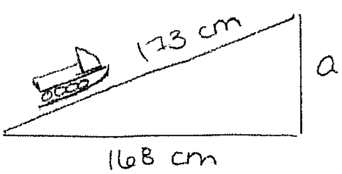
$h^2 = 3136 + 1089$

$\sqrt{h^2} = \sqrt{4225}$

$h = 65$ cm. ← h must be 65 cm in order to be a right triangle

∴ Since Marina's tabletop has a diagonal of 60 cm, it can not have square corners.

**Example 3: A ramp is used to load a snow machine onto a trailer. The ramp has horizontal length 168 cm and sloping length 173 cm. The side view is a right triangle. How high is the ramp?**




$h^2 = a^2 + b^2$

$(173)^2 = a^2 + (168)^2$

$29929 = a^2 + 28224$

$-28224 \qquad -28224$

$\sqrt{2401} = \sqrt{a^2}$

$49 = a$

∴ The ramp must be 49 cm high